7 An optimal rate of interest: the natural rate

7.1 Learning outcomes

After studying this text the learner should be able to:

1. Elucidate the Wicksell hypothesis.
2. Explain the significance and role of the natural rate.
3. Understand that there is a difference of opinion on the definition of the natural rate.
4. Discuss the essence of the Taylor rule and the place of the natural rate in the rule.

7.2 Introduction

Is there an optimal rate of interest? Before answering we need to make a diversion: and point out again the high correlation between DCE and GDP_N and the fact that it is an accepted fact in economics. It was covered in great detail in the seminal work of Friedman and Schwartz (1963 and 1982) (except that we see the link being between DCE and GDP_N – the effect on the money stock is the outcome of changes in bank lending / DCE):

> "…the movements in the level of [nominal] income and its rate of change parallel extraordinarily closely for more than a century the contemporaneous movements in the quantity of money and its rate of change, and this is equally true for the United States and the United Kingdom."

We show the relationship for yet another country[50] in Figure 7.1: the R^2 is 0.998. Why did we introduce this fact? It is because the division of GDP_N into its components real GDP (GDP_R) and inflation (P) is the most critical issue in economics. Friedman and Schwartz (1963 and 1982) paid much attention to it; in this regard they stated:

> "…the decomposition of nominal movements in money and income between prices and output. What determines this decomposition is one of the major unsettled questions in monetary theory; it has been the source of much of the dispute about the role of money in recent decades and is a key to the controversy between the quantity-theory framework and the Keynesian income-expenditure framework. It is the issue that generated the Phillips curve and all the opposing views that have peppered the economic literature. We do not, of course, settle these hotly debated issues, but we believe our data greatly clarify them."

The issue remains hotly debated and the final solution elusive, except that it is well-known that a low and stable ΔP is a propitious condition for achieving a ΔGDP_R at potential. The most favourable ΔP, based on extensive central banking experience and research, is 2.0% pa. As we know, 2.0% pa is the inflation target of most developed countries' central banks.

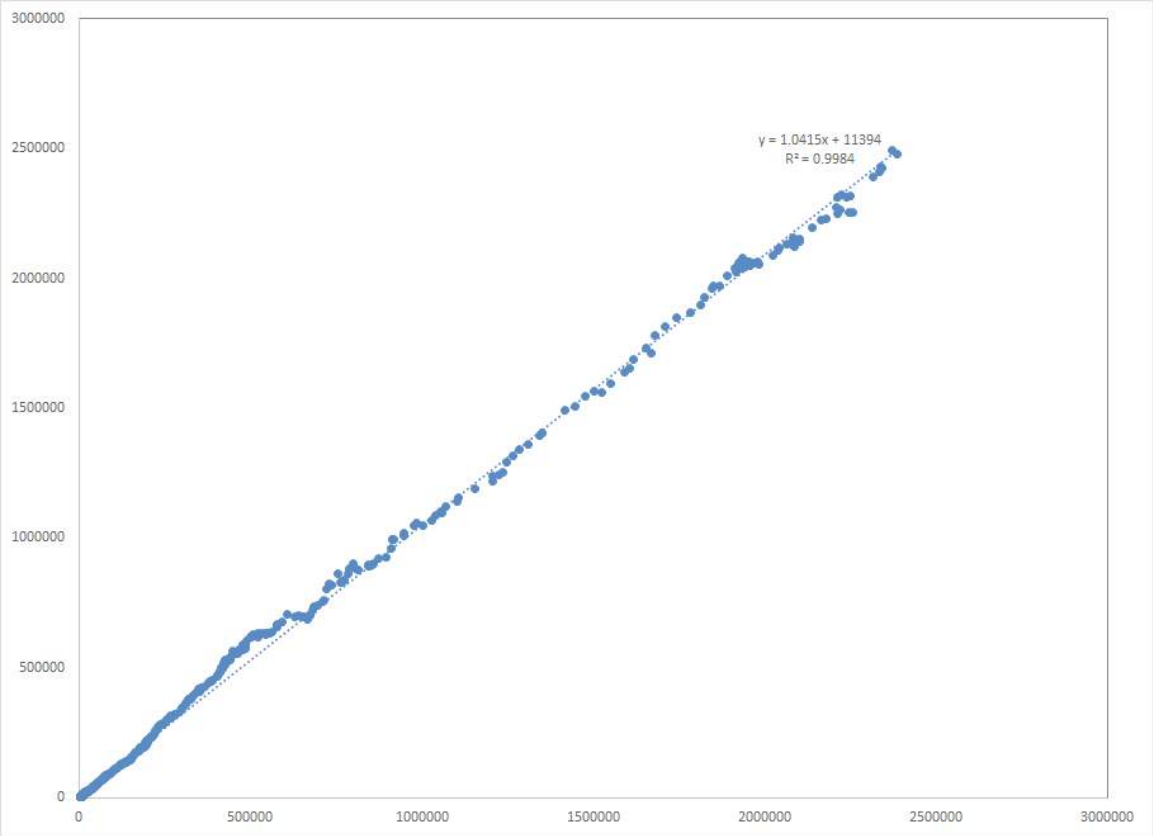

Figure 7.1: Scatter chart: M3 and DCE (raw monthly data)

With this as the backdrop, we are now able to answer the question: is there an optimal rate of interest? The answer is yes, and it is the "natural rate of interest", referred to here as the "natural rate" (NR), which can be defined as the equilibrium interest rate consistent with the optimal division of ΔGDP_N into its components ΔGDP_R and ΔP.

The NR was first presented to us in 1898 by the Swedish economist, Prof Knut Wicksell. However, the NR is not directly observable; it is hypothetical. The NR is a critical concept in that it is the rate (PIR or PR? – see below) which central banks aspire to in their quest to effectuate their mandate: low and stable inflation and high and sustainable economic output. We discuss the concept of the NR under the following headings:

- The Wicksell hypothesis.
- Literature review.
- An alternative interpretation.

- Reconciliation of PR and PIR / IBMR.
- Taylor rule.
- A proposed Taylor-type rule.

7.3 The Wicksell hypothesis

Prof Knut Wicksell famously wrote in 1898:

> "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them."

It is clear that Prof Wicksell embraced the fact that new money ($\Delta M3 = \Delta BD$) is created by new bank credit extended (Δbank credit) (substantiated below), and that there is a rate of interest, which he called the "natural rate" (note: a lending rate), which can bring about a state of equilibrium in the economy. This state is one where the division between ΔGDP_R and ΔP is such that ΔP (Wicksell's "commodity prices", in other words, inflation) is optimal: ΔP is low and stable and, against this background, ΔGDP_R is at the level of its long-term sustainable potential. In New Keynesian economic terms, there is no "output gap" [the divergence of ΔGDP_R from potential GDP_R growth (ΔGDP_{RP})], or the economy is at full employment, and inflation is low and stable.

If the economy is expanding at higher than GDP_{RP}, reflecting that, in simple terms, aggregate demand (AD) is outstripping aggregate supply (AS) and that higher inflation is or will the consequence, it means that the market rate is below the NR. Under these conditions, monetary policy will have to be tightened, that is, the PIR must be increased. The opposite also applies.

7.4 Literature review

Generally, scholars have interpreted the NR to be referenced on the PIR in real terms (PIR_R) or to the real Fedfunds rate, which is the US interbank market rate ($IBMR_R$). As we know, the IBMR is closely related to the PIR, as are their real rate versions. In some cases the NR is termed a "short-term rate" which, presumably, refers to the IBMR or the PIR. In a few cases the NR is referred to as just a "rate" without reference to the term. We present a few examples:

Federal Reserve Bank of San Francisco (2003):

> "…the natural rate is defined to be the real fed funds rate consistent with real GDP equaling [sic] its potential level (potential GDP) in the absence of transitory shocks to demand. Potential GDP, in turn, is defined to be the level of output consistent with stable price inflation, absent transitory shocks to supply. Thus, the natural rate of interest is the real fed funds rate consistent with stable inflation absent shocks to demand and supply.

Federal Reserve Bank of Cleveland (Carlstrom and Fuerst, 2003):

> "The second factor is the 'natural' real, or inflation-adjusted, federal funds interest rate. This is the rate that is consistent with 'neutral' monetary policy. That is, if the real funds rate is equal to the natural real rate, then monetary policy will be consistent with both the inflation and output targets."

Amato (2005):

> "Broadly put, the natural rate of interest…can be defined to be the equilibrium real interest rate consistent with price stability.… In the light of the key role that the natural rate plays, explicitly or implicitly, in theories of the transmission mechanism subscribed to by many central banks, estimates of the natural rate could prove useful to policy makers. Moreover, since most central banks formulate monetary policy by setting a target for a short-term nominal interest rate (typically an overnight money market rate), the natural rate provides a convenient benchmark that policy rates can be measured against directly."

Bank of Spain (Manrique and Marqués, 2004):

> "…direct comparison of the interest rate set by the central bank with the natural rate of interest has taken on greater importance in assessing the monetary policy stance."

Canzoneri, Cumby, Diba (2011):

> "Inflation is brought to its target if the policy rate is brought to its natural rate. Intuitively, when an increase in aggregate demand, or a decrease in productivity, pushes inflation above its target, the policy rate should be raised above its natural rate for a period of time, raising the real rate of interest to curb the rise in inflation."

Bank of France (Mésonnier and Renne, 2004):

> "…the natural rate of interest equals the equilibrium real rate of return in an economy where prices are fully flexible, or in other words, it is the real short term rate of interest that equates aggregate demand with potential output at all times.

Cuaresma, Gnan, Ritzberger-Gruenwald (2004):

> "…it is useful to define the natural rate of interest in terms of the real short-term interest rate where output converges to potential and inflation is stable…"

Laidler (2011):

> "Natural rate of interest: The value of an economy-wide average of the real rates of interest at which agents transact that equates the economy-wide supply of savings to the economy-wide demand for investment, and hence ensures that the output gap is zero."

Sveriges Riksbank (Central Bank of Sweden) (Lundvall and Westermark, 2011):

> "The natural interest rate is the real interest rate that would prevail if resource utilisation in the economy was normal today and was expected to remain normal in the future."

Thus, according to the literature, the majority of scholars regard the NR as the ideal or optimal PIR_R or $IBMR_R$, that is, the rate level that is consistent with both low and stable inflation (ΔP) and potential output (ΔGDP_{RP}). It may be regarded as the optimal division of ΔGDP_N into its components.

The literature is ad idem on the issue of the NR not being a constant rate. It shifts with changing economic circumstances. Much attention has been devoted by many scholars to estimating the changing NR, called the "time-varying natural rate of interest" (TVNRI). One study (Cuaresma, Gnan, Ritzberger-Gruenwald, 2004) reported that for the euro area the NR: "…now fluctuates between 1 and 3½% between 1994 and spring 2002. The average over the full sample period is close to 3%."

Taylor, of Taylor rule fame (see below), estimated the average for the US at 2.0%.

7.5 An alternative interpretation

We are of the opinion that the scholars mentioned have misinterpreted Wicksell. In substantiation, we repeat the Wicksell quote from above (1898) (bold: the author's):

> "There is a certain **rate of interest on loans** which is neutral in respect to commodity prices, and tends neither to raise nor to lower them."

Further substantiation (Wicksell, 1898) (italics: sic; bold: the author's):

> "This does not mean that the **banks** ought actually to ascertain the natural rate before fixing their **own rates of interest**. That would, of course, be impracticable, and would also be quite unnecessary. For the current level of commodity prices provides a reliable test of the agreement or diversion of the two rates. The procedure should rather be simply as follows*: So long as prices remain unaltered the **banks' rate of interest** is to remain unaltered. If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of prices calls for a further change in one direction or the other.*"

We also add Wicksell's (1907) consideration of money creation resulting from bank credit extended (bold: the author's):

> "The **banks in their lending business** are not only not limited by their own capital; they are not, at least not immediately, limited by any capital whatever; by concentrating in their hands almost all payments, they themselves create the money required, or, what is the same thing, they accelerate *ad libitum* the rapidity of the circulation of money. The sum borrowed today in order to buy commodities is placed by the seller of the goods on his account at the same bank or some other bank… Emil Struck, justly says in his well-known sketc.h of the English money market: in our days demand and supply of money have become about the same thing, the demand to a large extent creating its own supply. In a *pure* system of credit, where all payments were made by transference in the bank-books, the **banks** would be able to grant at any moment any amount of **loans** at any…**rate of interest**."

Contemplation of the highlighted (in bold) parts will reveal that Wicksell was referring to the banks' lending rate/s, which we have called prime rate (PR), the benchmark bank lending rate. This is in line with our analysis presented above, that is, that the level of PR in real terms (PR_R) is the rate which, to a large degree, determines the demand for bank credit and its outcome, deposit money creation. Additional aggregate demand in nominal terms [$\Delta(C + I) + \Delta NE = \Delta GDP_N$] (NE = net exports) underlies the bank-satisfied demand for credit. It is axiomatic that households, companies and government borrow (and they do so to a large degree from the banks) to spend (C) or invest (I).

We presented the relationship (DCE and GDP_N) in the case of the US earlier ($R^2 = 0.98$), and present it for Japan ($R^2 = 0.99$) and Switzerland ($R^2 = 0.99$) in Figures 7.2–7.3 (World Bank data). (See also later for additional data.)

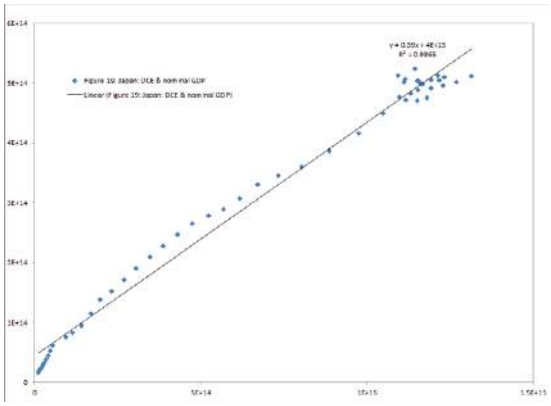

Figure 7.2: Japan: DCE & nominal GDP

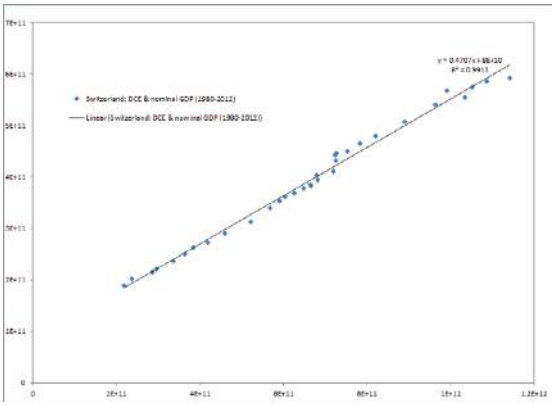

Figure 7.3: Switzerland: DCE & nominal GDP

Prof Wicksell's (1898) (bold: the author's) strong emphasis on inflation is notable and he speaks of an inflation-targeting monetary policy regime, much like we see today; while he did not specify the level of inflation, it is clear that he had a low rate in mind:

> *"…so long as **prices** remain unaltered the banks' rate of interest is to remain unaltered. If **prices** rise, the rate of interest is to be raised; and if **prices** fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of **prices** calls for a further change in one direction or the other."*

It can be safely assumed that Prof Wicksell was referring to the banks' *lending rate* (the modern day PR), and it can be surmised that he meant the level of PR_R. Thus, he was referring to the "optimal" PR_R as the NR, i.e. the hypothetical rate that would produce the best outcome in terms of inflation and real output. This ideal outcome has been grappled with by the best academic and central bank minds to this day.

7.6 Reconciliation of PR and PIR / IBMR

Thus, there is a difference of opinion with regard to the reference rate for the NR. However, we also know that there is (in most countries) a close correlation between the PIR / IBMR and the PR (as a reminder, see Figures 7.5–7.6). The data are monthly for a period of over 50 years for a particular country[51]. The R^2 is 0.98, and it is not a spurious one. Causation is (as discussed earlier):

> PIR → IBMR → call money rates (wholesale) → other deposit rates → [via the sticky bank margin] → PR.

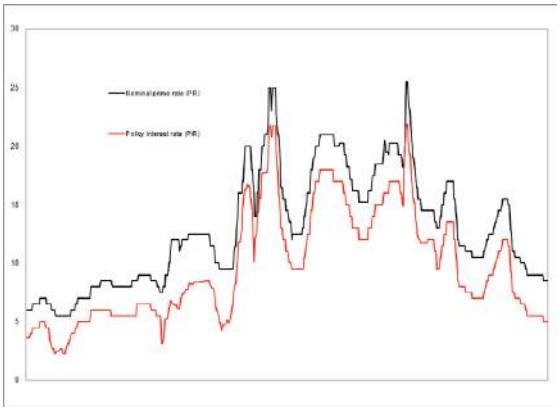

Figure 7.5: PIR and PR (50 years)

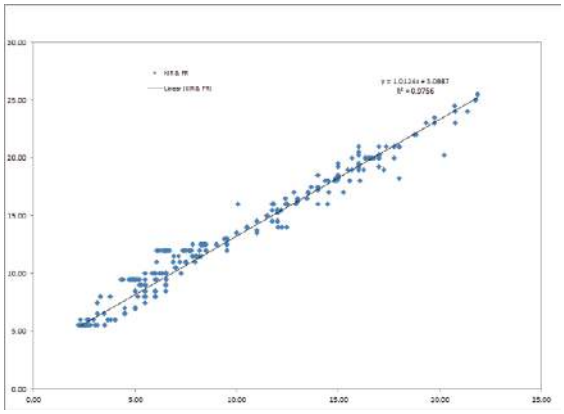

Figure 7.6: Scatter chart: PIR and PR (50 years)

The changes (PIR → PR) are one-for-one, as they are in real terms as well. However, there is a major difference in the *levels* of PIR and PR, and the level of PR is crucial in the influence on the demand for credit / money creation (when demand is satisfied), which, as we has said, reflects additional demand / output.

If we accept that:

(1) bank credit extended creates bank new deposits, i.e. money (which is axiomatic),
(2) that bank credit growth (ΔDCE) largely reflects the growth in aggregate demand (ΔAD),
(3) that, given (1) and (2), the PR is the reference rate for the NR,
(4) and that a static margin between PIR and PR exists (there is evidence that it does, and it is in the region of 3.0 percentage points),

then it is important for monetary policy to establish the PR, and thereby the PR_R, at the optimal level (= NR). It is clear that in order to do so, the PIR must be at a level that achieves this. As we know, central banks have the tools to do so.

If, at a particular level, the PR_R is consistent with the ideal division between ΔP and ΔGDP_R, then PR_R = NR. Then, if inflation moves above the target level (as said, the world seems to have accepted a 2% target as ideal), it will be recognised that, to a large degree, additional aggregate demand (ΔAD) was fuelled by credit / money creation, and that aggregate supply (ΔAS) could not adjust quickly enough (a low elasticity of supply). The bank credit / money creation data should reflect this.

Thus, it will be known that PR_R < NR and that PR and PR_R must be adjusted upwards, by adjusting the PIR upwards (to what extent we discuss in the following section on the Taylor principle). Conversely, if inflation is below target or deflation is foreseen, then PR_R > NR, and it must be adjusted downwards. Once again the PIR is the route to the PR.

In conclusion, it will be apparent that central banks have great difficulty in establishing when PR_R = NR. Their mandate is to keep inflation low and stable (in developed countries 2.0%) because history has shown that an inflation rate of 2.0% presents the ideal condition for achievement of potential ΔGDP_R. Non-achievement of PR_R = NR is a common occurrence with the majority of central banks, indicating the difficulty of establishing when PR_R = NR. There also exists the problem of a shifting NR.

This does not mean that the search for the NR should be abandoned. Rather, efforts to approximate it should continue with vigour.

7.7 Taylor rule

The Taylor rule is relevant to this discussion. It is a monetary policy rule which links mechanically the level of the PIR to:

- deviations of inflation from its target level, and
- deviations of output from its potential level (the output gap).

In other words it focusses on rectifying, through changes in the PIR, deviations from the optimal outcome of the NR: low and stable inflation and sustainable high real growth. The Taylor (1993) rule (Hofmann, Bogdanova, 2012 interpretation) is as follows:

$$PIR = (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*)$$

where

PIR = target nominal policy interest rate
r^* = the long-run or equilibrium real rate of interest (i.e. the NR)
π^* = central bank's inflation target
π = current period inflation rate
y = current period GDP_R growth
y^* = long term GDP_R growth (= potential growth rate).

It will be clear that $(y - y^*)$ is the output gap, that is, the deviation of current GDP_R growth from the long-term average, which can be regarded as the potential growth rate.

An example where $\pi^* = \pi$, and $y = y^*$:

r^* = 2.0% (the rate favoured / assumed by Prof Taylor)
π^* = 2.0% (the rate favoured by central banks in developed countries)
π = 2.0% (assumed)
y = 2.5% (assumed)
y^* = 2.5% (assumed)

$$\begin{aligned} PIR &= (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*) \\ &= (2.0 + 2.0) + [1.5 \times (2.0 - 2.0)] + [0.5 \times (2.5 - 2.5)] \\ &= 4.0 + 0 + 0 \\ &= 4.0\% \text{ pa.} \end{aligned}$$

In other words, the $PIR_N = r^* + \pi^*$, that is, the NR + the current inflation rate. An example of a booming economy with inflation above target:

r^* = 2.0% (the rate favoured / assumed by Prof Taylor)
π^* = 2.0% (the rate favoured by central banks in developed countries)
π = 3.0% (assumed)
y = 3.5% (assumed)
y^* = 2.5% (assumed)

$$\begin{aligned} \text{PIR} &= (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*) \\ &= (2.0 + 2.0) + [1.5 \times (3.0 - 2.0)] + [0.5 \times (3.5 - 2.5)] \\ &= 4.0 + 1.5 + 0.5 \\ &= 6.0\% \text{ pa.} \end{aligned}$$

It will be clear that the Taylor rule prescribes a "leaning against the wind" policy, that is, it "…implies that central banks aim at stabilising inflation around its target level and output around its potential. Positive (negative) deviations of the two variables from their target or potential level would be associated with a tightening (loosening) of monetary policy." (Hofmann, Bogdanova, 2012.) It will also be apparent that a change in inflation is met with a change in the nominal PIR that *exceeds* the change in inflation. This is referred to as the "Taylor principle".

The Taylor rule is a highly esteemed piece of work, but is not without limitations. Taylor himself accepts that circumstances do prevail (referred to as "shocks" in the literature) when central banks may be required to deviate from the rule. It is also well known that the rule involves assumptions about the non-observable NR, which is a shifting rate. It may at times therefore be incorrect and misleading.

7.8 A proposed Taylor-type rule

It is clear where the NR fits into the Taylor rule: r*. But, what about our view that the reference rate of the NR should be the PR_R and not the PIR_R? As such, we are of the opinion that the PR_N should be the target interest rate (it is in many countries) and that the PIR_N should be derived from it as follows:

$$PIR_N = PR_N - (\text{long-term differential: } PR_N - PIR_N).$$

Why do we believe this is significant? It is because:

- To a significant degree ΔDCE = ΔM3, and the latter is the outcome of the former. The R2 (World Bank data):
  - Raw data: USA = 0.996, UK = 0.994, Canada = 0.97, Australia = 0.996, Japan = 0.981, Switzerland = 0.97, South Africa (monthly central bank data) = 0.9992.
  - Yoy% changes: USA 0.7, UK = 0.64, Canada = 0.83, Australia = 0.7, Japan = 0.89, South Africa = 0.7.

- To a large degree ΔDCE reflects changes in ΔGDP_N, and therefore changes in aggregate demand (ΔAD). The R2 (World Bank data):
  - Raw data: USA = 0.98, UK = 0.92, Canada = 0.91, Australia = 0.94, Japan = 0.99, Switzerland = 0.99, South Africa = 0.99.
  - Yoy% changes: as in above case the numbers are lower but robust: USA = 0.5, UK (we suspect data problems), Canada = 0.4, Australia = 0.2, Japan = 0.6, Switzerland (data problems), South Africa = 0.23.

- The level of PR in nominal and real terms is the main ingredient in changes in ΔDCE,
- There is a one-to-one relationship between ΔPIR and ΔPR.

It is perhaps necessary to create a Taylor-type rule for PR:

$$PR_N = (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*)$$

where

PR_N = target bank prime lending rate.
r^* = the long-run or optimal / equilibrium real PR (i.e. the proposed NR)
π^* = central bank's inflation target
π = current period inflation rate
y = current period GDP_R growth
y^* = long term GDP_R growth (= potential growth rate).

An example where $\pi^* = \pi$, and $y = y^*$:

r^* = 3.5% (based on a period in a developing country[52] when PR_R delivered a high and sustainably high ΔGDP_R = 5.5% and a low ΔP = 3.0%, i.e. an approximate NR in our opinion)
π^* = 3.0% (assumed target inflation rate favoured by central banks in developing countries)
π = 3.0% (assumed)
y = 5.5% (assumed)
y^* = 5.5% (assumed)

$$\begin{aligned} PR_N &= (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*) \\ &= (3.5 + 3.0) + [1.5 \times (3.0 - 3.0)] + [0.5 \times (5.5 - 5.5)] \\ &= 6.5 + 0 + 0 \\ &= 6.5\% \text{ pa.} \end{aligned}$$

In other words, the $PR_N = r^* + \pi^*$, i.e. the NR + the current inflation rate. If the long-run differential between PR_N and PIR_N = 3.0 percentage points, then the derived PIR_N:

$$\begin{aligned} PIR_N &= PR_N - (\text{long-run } PR_N - PIR_N) \\ &= 6.5 - 3.0 \\ &= 3.5\% \text{ pa.} \end{aligned}$$

An example of a booming economy with inflation above target:

r^* = 3.5% (as above)
π^* = 3.0% (the rate favoured by central banks in developing countries)
π = 6.0% (assumed)
y = 6.5% (assumed)
y^* = 5.5% (assumed)

$$\begin{aligned} PR_N &= (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*) \\ &= (3.5 + 3.0) + [1.5 \times (6.0 - 3.0)] + [0.5 \times (6.5 - 5.5)] \\ &= 6.5 + (1.5 \times 3.0) + (0.5 \times 1.0) \\ &= 6.5 + 4.5 + 0.5 \\ &= 11.5\% \text{ pa.} \end{aligned}$$

The PIR_N then becomes 11.5 – 3.0 = 8.5% pa.

It is notable that with this approach (NR = PR_R and not PIR_R), the translation into the PIR is amplified (compared with the Taylor rule outcome), and this is because a change to the PIR is not the same, percentage-wise, as a change in PR. For example if PIR = 4.0% pa and is increased to 4.5%, this = 12.5%. If PR = 8.0% and changes in line with PIR to 8.5%, this = 6.25%. This means that when, for example, inflation and output rise above the target levels, the monetary policy reaction (change in PIR) will be harsher than that dictated by the Taylor rule.

Let us conclude with an example of an underperforming economy with low inflation:

r^* = 3.5% (as above)
π^* = 3.0% (the rate favoured by central banks in developing countries)
π = 2.0% (assumed)
y = 2.5% (assumed)
y^* = 5.5% (assumed)

$$\begin{aligned} PR_N &= (r^* + \pi^*) + 1.5(\pi - \pi^*) + 0.5(y - y^*) \\ &= (3.5 + 3.0) + [1.5 \times (2.0 - 3.0)] + [0.5 \times (2.5 - 5.5)] \\ &= 6.5 + (1.5 \times -1.0) + (0.5 \times -3.0) \\ &= 6.5 - 1.5 - 1.5 \\ &= 3.5\% \text{ pa.} \end{aligned}$$

The PIR_N then becomes 3.5 – 3.0 = 0.5% pa. It is notable that this level fits with a QE-type policy.

In summary: the proposal is for the PR_R, not the PIR_R, to be the reference rate of the NR. If PR_R = NR, then monetary policy, as reflected in the derived PIR_N, is consistent with the output and inflation targets. It is recognised that times do arise when short-term deviations may be required.

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